

Charmonium Mass Spectrum with Spin-Dependent Interaction in Momentum-Helicity Space

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In this paper we have solved the nonrelativistic form of the Lippmann-Schwinger equation in the momentum-helicity space by inserting a spin-dependent quark-antiquark potential model numerically. To this end, we have used the momentum-helicity basis states for describing a nonrelativistic reduction of one gluon exchange potential. Then we have calculated the mass spectrum of the charmonium $\psi(c\bar{c})$, and finally we have compared the results with the other theoretical results and experimental data.

1. INTRODUCTION

During the past years, several models and methodological approaches based on solving the relativistic and nonrelativistic form of the Schrödinger or Lippmann-Schwinger equation have been developed for studying the light and heavy mesons in the coordinate and momentum spaces respectively.

Recently, the three-dimensional approach based on momentum-helicity basis states for studying the Nucleon-Nucleon scattering and deuteron state has been developed [1, 2]. We extend this approach to particle physics problems by solving the nonrelativistic form of the Lippmann-Schwinger equation to obtain the mass spectrum of the heavy mesons using the nonrelativistic quark-antiquark interaction in terms of a linear confinement, a Coulomb, and various spin-dependent pieces.

In the heavy-quark (c,b) mesons the differences between energy levels are small compared to the particle masses. Hence, the nonrelativistic Lippmann-Schwinger equation can be used to study their quantum behavior. To this end, we have used the nonrelativistic form of the Lippmann-Schwinger equation in the momentum-helicity representation to study the charmonium as a heavy meson. For this purpose, we have used a nonrelativistic quark-antiquark potential based on one-gluon exchange in the momentum-helicity representation.

This article is organized as follows. In Sect. 2, the nonrelativistic Lippmann-Schwinger equation in the momentum-helicity basis states which leads to coupled and uncoupled integral equations for various quantum numbers is presented briefly. In Sect. 3, a spin dependent quark-antiquark potential model is described in the momentum-helicity basis states. The details of the numerical calculations and the results obtained for the charmonium are presented in Sect. 4. Finally, a summary and an outlook are provided in Sect. 5.

2. LIPPMANN-SCHWINGER EQUATION IN MOMENTUM-HELICITY BASIS STATES

The nonrelativistic form of the homogenous Lippmann-Schwinger equation for describing the heavy meson bound state is given by:

$$|\Phi_j^{M_j}\rangle = \frac{1}{E - \frac{p^2}{m}} V |\Phi_j^{M_j}\rangle, \quad (1)$$

where V denotes the quark-antiquark interaction, m is mass of the quark or antiquark and $|\Phi_j^{M_j}\rangle$ is the meson bound state with the total angular momentum j . M_j is projection of the total angular momentum j along the quantization axis. The integral form of this equation in the momentum-helicity basis states is written as [3]:

$$\Phi_{Sj}^{M_j}(p) = \frac{2\pi}{E - \frac{p^2}{m}} \sum_{\Lambda'} \int_0^\infty dp' p'^2 V_{M_j\Lambda'}^S(p, p') \Phi_{Sj}^{\Lambda'}(p'), \quad (2)$$

with:

$$V_{M_j\Lambda'}^S(p, p') = \int_{-1}^1 d\cos\theta' V_{M_j\Lambda'}^S(p, p', \theta') d_{M_j\Lambda'}^j(\theta'), \quad (3)$$

where p is the magnitude of the relative momentum of the quark and antiquark, S is the total spin of meson, Λ is the spin projection along the relative momentum and $d_{M_j\Lambda'}^j(\theta')$ are the rotation matrices. For an arbitrary total angular

momentum j , and singlet case of the total spin state, Eq. (2) leads to one equation:

$$\Phi_{0j}^{M_j}(p) = \frac{2\pi}{E - \frac{p^2}{m}} \int_0^\infty dp' p'^2 V_{M_j 0}^0(p, p') \Phi_{0j}^0(p'). \quad (4)$$

Also for $j = 0$ and triplet case of the total spin state, Eq. (2) leads to one equation as:

$$\Phi_{1j}^0(p) = \frac{2\pi}{E - \frac{p^2}{m}} \int_0^\infty dp' p'^2 V_{00}^1(p, p') \Phi_{1j}^0(p'). \quad (5)$$

For $S = 1$ and $j > 0$ it is more complicated. For example for $j = 1$, Eq. (2) leads to one equation for channel P and two coupled equations for channels S and D as follows:

$$\Psi_{111}(p) = \frac{2\pi}{E - \frac{p^2}{m}} \int_0^\infty dp' p'^2 \left[V_{11}^1(p, p') - V_{-11}^1(p, p') \right] \Psi_{111}(p'), \quad (6)$$

$$\begin{aligned} \Psi_{011}(p) = & \frac{2\pi}{E - \frac{p^2}{m}} \frac{1}{3} \int_0^\infty dp' p'^2 \left\{ \left[2V_{11}^1(p, p') + 2V_{01}^1(p, p') + V_{00}^1(p, p') + 2V_{10}^1(p, p') + 2V_{-11}^1(p, p') \right] \Psi_{011}(p') \right. \\ & \left. + \sqrt{2} \left[V_{11}^1(p, p') + V_{01}^1(p, p') - V_{00}^1(p, p') - 2V_{10}^1(p, p') + V_{-11}^1(p, p') \right] \right\} \Psi_{211}(p'), \end{aligned} \quad (7)$$

$$\begin{aligned} \Psi_{211}(p) = & \frac{2\pi}{E - \frac{p^2}{m}} \frac{1}{3} \int_0^\infty dp' p'^2 \left\{ \sqrt{2} \left[V_{11}^1(p, p') - 2V_{01}^1(p, p') - V_{00}^1(p, p') + V_{10}^1(p, p') + V_{-11}^1(p, p') \right] \Psi_{011}(p') \right. \\ & \left. + \left[V_{11}^1(p, p') - 2V_{01}^1(p, p') + 2V_{00}^1(p, p') - 2V_{10}^1(p, p') + V_{-11}^1(p, p') \right] \right\} \Psi_{211}(p'), \end{aligned} \quad (8)$$

where $\Psi_{lSj}(p)$ is the partial wave component of the wave function which is connected to the momentum-helicity component of the wave function as [3]:

$$\Phi_{jS}^\Lambda(p) = \sum_l \sqrt{\frac{2l+1}{4\pi}} C(lsj; 0\Lambda\Lambda) \Psi_{lSj}(p). \quad (9)$$

The inverse relation is written as:

$$\Psi_{lSj}(p) = \frac{\sqrt{4\pi(2l+1)}}{2j+1} \sum_\Lambda C(lsj; 0\Lambda\Lambda) \Phi_{jS}^\Lambda(p). \quad (10)$$

3. QUARK-ANTIQUARK POTENTIAL IN MOMENTUM-HELICITY BASIS STATES

The spin dependent potential model that we have used in our calculations is sum of the Linear and a simple nonrelativistic reduction of an effective one gluon exchange potential without retardation. This potential in the coordinate space is given in terms of [4]:

$$\begin{aligned} V(\mathbf{r}, \mathbf{p}) = & \sigma r + f_c \alpha_s \left\{ \frac{1}{r} - \frac{\pi}{m^2} \delta(\mathbf{r}) + \frac{1}{m^2} \frac{\mathbf{p} \cdot \mathbf{p}}{r} - \frac{3}{4m^2} \frac{\mathbf{L} \cdot (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)}{r^3} \right. \\ & \left. - \frac{2\pi}{3m^2} \delta(\mathbf{r}) (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) - \frac{1}{4m^2} \frac{3(\boldsymbol{\sigma}_1 \cdot \hat{\mathbf{r}})(\boldsymbol{\sigma}_1 \cdot \hat{\mathbf{r}}) - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)}{r^3} \right\}, \end{aligned} \quad (11)$$

where σ is the string tension, α_s is the strong-interaction fine-structure constant, f_c is the color factor which is $-4/3$ for quark-antiquark and $-2/3$ for quark-quark, $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are the Pauli matrices and \mathbf{L} is the total orbital angular

momentum operator. Fourier transformation of this potential to momentum space yields:

$$\begin{aligned} \langle \mathbf{p} | V | \mathbf{p}' \rangle = & \sigma \left[\delta(\mathbf{q}) r_c + \frac{1}{2\pi^2 q^4} (2 \cos(q r_c) - 2 + q r_c \sin(q r_c)) \right] \\ & + f_c \alpha_s e^{-\lambda^2 q^2} \left\{ \left[\frac{\delta(\mathbf{q})}{r_c} + \frac{1}{2\pi^2 q^2} \left(1 - \frac{\sin(q r_c)}{q r_c} \right) \right] \left(1 + \frac{p^2}{m^2} \right) - \frac{1}{8\pi^2 m^2} \right. \\ & \left. + \frac{3}{8\pi^2 m^2 q^2} i(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \mathbf{p} \times \mathbf{p}' - \frac{1}{12\pi^2 m^2} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + \frac{1}{24\pi^2 m^2} [3(\boldsymbol{\sigma}_1 \cdot \hat{\mathbf{q}})(\boldsymbol{\sigma}_2 \cdot \hat{\mathbf{q}}) - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)] \right\}, \quad (12) \end{aligned}$$

where $\mathbf{q} = \mathbf{p}' - \mathbf{p}$ is the momentum transfer. The kernels of integral equations have singularity. To overcome this problem we have used the regularized form of linear confining and Coulomb parts of the potential [5]. Details of Fourier transformation of regularized parts of the potential are given in Appendix A. Also we have used a Gaussian form factor, $\exp(-\frac{1}{2}\lambda^2 q^2)$ at the quark-gluon vertex as in Ref. [6] to remove singularity of the kernels due to existence of one gluon exchange potential. The variable λ can be interpreted as size of the quark. In Ref. [7] the pointlike quark-gluon vertex is replaced by a form factor, $1/(q^2 + \beta^2)$ in which β^{-1} is the effective quark size to eliminate the singularity. In this work we have used both regularized form and Gaussian form factor for coulomb and $f_c \alpha_s \mathbf{p}^2/(m^2 r)$ parts of the potential which cause the convergence of numerical results faster. Therefore, the final form of the potential in the momentum-helicity space is written as:

$$\begin{aligned} V_{\Lambda\Lambda'}^S(\mathbf{p}, \mathbf{p}') \equiv \langle \mathbf{p} S \Lambda | V | \mathbf{p}' S \Lambda' \rangle = & \sigma \langle \hat{\mathbf{p}} S \Lambda | \hat{\mathbf{p}}' S \Lambda' \rangle \left[\delta(\mathbf{q}) r_c + \frac{1}{2\pi^2 q^4} (2 \cos(q r_c) - 2 + q r_c \sin(q r_c)) \right] \\ & + f_c \alpha_s e^{-\lambda^2 q^2} \langle \hat{\mathbf{p}} S \Lambda | \hat{\mathbf{p}}' S \Lambda' \rangle \left\{ \frac{1}{r_c} \delta(\mathbf{q}) + \frac{1}{2\pi^2 q^2} \left(1 - \frac{\sin(q r_c)}{q r_c} \right) \left(1 + \frac{p^2}{m^2} \right) \right. \\ & - \frac{1}{8\pi^2 m^2} - \frac{1}{12\pi^2 m^2} (2S(S+1) - 3) \\ & + \frac{3}{8\pi^2 m^2} \frac{p p'}{q^2} \left[\gamma S(S+1) - 2\Lambda\Lambda' - \frac{1}{\gamma} (S(S+1) - 2\Lambda'^2 - 2\Lambda^2 - 2\Lambda'^2 \Lambda^2) \right] \\ & - \frac{1}{24\pi^2 m^2 q^2} [6p p' \Lambda\Lambda' + 2p'^2 (S(S+1) - 3\Lambda'^2) + 2p^2 (S(S+1) - 3\Lambda^2) \\ & \left. - p p' \gamma S(S+1) - 3 \frac{p p'}{\gamma} (S(S+1) - 2\Lambda'^2 - 2\Lambda^2 - 2\Lambda'^2 \Lambda^2) \right] \Big\}, \quad (13) \end{aligned}$$

where $\gamma = \hat{\mathbf{p}}' \cdot \hat{\mathbf{p}} = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\varphi - \varphi')$ and $|\mathbf{p}; \hat{\mathbf{p}} S \Lambda\rangle$ is the momentum-helicity basis state which is eigenstate of the helicity operator $\mathbf{S} \cdot \hat{\mathbf{p}}$ as:

$$\mathbf{S} \cdot \hat{\mathbf{p}} |\mathbf{p}; \hat{\mathbf{p}} S \Lambda\rangle = \Lambda |\mathbf{p}; \hat{\mathbf{p}} S \Lambda\rangle. \quad (14)$$

Also we have [1]:

$$\langle \hat{\mathbf{p}} S \Lambda | \hat{\mathbf{p}}' S \Lambda' \rangle = \sum_{N=-S}^S e^{iN(\varphi-\varphi')} d_{N\Lambda}^S(\theta) d_{N\Lambda'}^S(\theta'). \quad (15)$$

If the vector \mathbf{p} is along z -direction, it is clear that the Eq. (15) is reduced to:

$$\langle \hat{\mathbf{z}} S \Lambda | \hat{\mathbf{p}}' S \Lambda' \rangle = e^{-i\Lambda\varphi'} d_{\Lambda\Lambda'}^S(\theta'). \quad (16)$$

For numerical calculations we need the matrix elements of the potential $V_{\Lambda\Lambda'}^S(p, p', \theta')$. These matrix elements is related to the matrix elements Eq. (13) as follows:

$$V_{\Lambda\Lambda'}^S(p, p', \theta') = e^{i\Lambda\varphi'} \langle p \mathbf{z}; \hat{\mathbf{z}} S \Lambda | V | \mathbf{p}'; \hat{\mathbf{p}}' S \Lambda' \rangle. \quad (17)$$

By considering Eqs. (13), (16) and (17), the final form of the matrix elements of the potential which is inserted in the numerical calculations is written as:

$$V_{\Lambda\Lambda'}^S(p, p', \theta') = \sigma d_{\Lambda\Lambda'}^S(\theta') \left[\delta(\mathbf{q}) r_c + \frac{1}{2\pi^2 q^4} (2 \cos(q r_c) - 2 + q r_c \sin(q r_c)) \right]$$

$$\begin{aligned}
& + f_c \alpha_s e^{-\lambda^2 q^2} d_{\Lambda\Lambda'}^S(\theta') \left\{ \frac{1}{r_c} \delta(\mathbf{q}) + \frac{1}{2\pi^2 q^2} \left(1 - \frac{\sin(q r_c)}{q r_c} \right) \left(1 + \frac{p^2}{m^2} \right) \right. \\
& - \frac{1}{8\pi^2 m^2} - \frac{1}{12\pi^2 m^2} (2S(S+1) - 3) \\
& + \frac{3}{8\pi^2 m^2} \frac{p p'}{q^2} \left[\gamma S(S+1) - 2\Lambda\Lambda' - \frac{1}{\gamma} (S(S+1) - 2\Lambda'^2 - 2\Lambda^2 - 2\Lambda'\Lambda^2) \right] \\
& - \frac{1}{24\pi^2 m^2 q^2} \left[6p p' \Lambda\Lambda' + 2p'^2 (S(S+1) - 3\Lambda'^2) + 2p^2 (S(S+1) - 3\Lambda^2) \right. \\
& \left. \left. - p p' \gamma S(S+1) - 3 \frac{p p'}{\gamma} (S(S+1) - 2\Lambda'^2 - 2\Lambda^2 - 2\Lambda'\Lambda^2) \right] \right\}, \tag{18}
\end{aligned}$$

with $\gamma = \hat{\mathbf{p}}' \cdot \hat{\mathbf{z}} = \cos \theta'$.

4. DISCUSSION AND NUMERICAL RESULTS

For numerical calculations as a first step we have used the Gaussian quadrature grid points to discretize the momentum and the angle variables. The integration interval for the momentum is covered by two different hyperbolic and linear mappings of the Gauss-Legendre points from the interval $[-1, +1]$ to the intervals $[0, p_2] \cup [p_2, p_{max}]$ respectively as:

$$p = \frac{1+x}{\frac{1}{p_1} + \left(\frac{2}{p_2} - \frac{1}{p_1}\right)x}, \quad p = \frac{p_{max} - p_2}{2}x + \frac{p_{max} + p_2}{2}. \tag{19}$$

Then we have calculated the matrix elements of the potential $V_{\Lambda\Lambda'}(p, p', \theta')$, from Eq. (18). According to the Eq. (3) integration over the spherical angle variable θ' , has been done independently. Finally, we have solved the integral equations (4)-(8) as eigenvalue equations. The integration over momentum variable is cut off at $q_{max} = 10$ GeV. This selection is carried out so that the numerical results do not depend on this choice. The typical values for p_1 and p_2 are 1 GeV and 3 GeV, respectively. These selections are done till the total number of grid points for momentum intervals are decreased. Other selections can be done but by different grid points for momentum variables.

The parameters of the potential model which are shown in Table I are fixed by a fit to the masses of the states η_c , J/ψ and h_c , similar to what is done in Ref. 9. The results of charmonium mass spectrum are shown in Table II. They are compared with the experimental data and another theoretical work. From Eqs. (7) and (8) it is clear that existence of the tensor term in the potential mix S - and D - partial waves but this mixed as it is shown in Table III is so weak. I show the mixed charmonium states in Table II by their dominant partial wave.

As a test of our numerical calculations we have shown convergence of the results as a function of number of grid points N_{P1} , N_{P2} and N_θ for the momentum and angle variables in Table IV. N_{P1} , N_{P2} are the number of grid points for the intervals $[0, p_2]$ and $[p_2, p_{max}]$ respectively. N_θ is corresponding to number of grid points for spherical angle variable. In our calculations we have chosen $N_{P1}=100$, $N_{P2}=100$ $N_\theta=200$ grid points for to achieve an acceptable accuracy.

5. SUMMARY AND OUTLOOK

In this paper we have extended an approach based on momentum-helicity basis states for calculation of mass spectrum of heavy mesons by solving nonrelativistic form of the Lippmann-Schwinger equation. As an application we have used this approach to obtain the mass spectrum of charmonium. The advantage of working with helicity states is that states are the eigenstates of the helicity operator appearing in the quark-antiquark potential. Thus, using the helicity representation is less complicated than using the spin representation with a fixed quantization axis for representation of spin dependent potentials. This work is the first step toward for studying single, double, and triple heavy-flavor baryons in the framework of the nonrelativistic quark model by formulation of the Faddeev equation in the 3D momentum-helicity representation. Furthermore, we can apply this formalism straightforwardly for investigation of heavy pentaquark systems, which can be considered as two-body (heavy meson, baryon) systems with meson-nucleon potentials which is underway.

TABLE I: Parameters of the model.

σ [GeV/fm]	1.222
λ [GeV ⁻¹]	0.3154
m [GeV]	1.269
α_s	0.2863
r_c [fm]	10

TABLE II: Comparison of the obtained charmonium mass spectrum with the experimental data and another work.

$n^{2S+1}L_J$	Candidate	Exp. [8]	Ref. [9]	Mass [MeV]
1^1S_0	η_c	2980.4 ± 1.2	2980	2980.4
1^3S_1	J/ψ	3096.916 ± 0.011	3097	3096.9
1^1P_1	h_c	3526.21 ± 0.25	3527	3526.2
1^3P_0	χ_{c0}	3415.16 ± 0.35	3430	3397.4
1^3P_1	χ_{c1}	3510.59 ± 0.10	3503	3503.5
2^1S_0	η'_c	3638 ± 5	3674	3683.1
2^3S_1	ψ'	3686.093 ± 0.034	3765	3760.8
1^3D_1	ψ''	3770 ± 2.4	3855	3850.6
3^3S_1	ψ'''	4040 ± 10	4291	4285.4

TABLE III: Percent of each partial wave in mixed charmonium states.

$n^{2S+1}L_J$	$c\bar{c}$	$P_S\%$	$P_D\%$
$1^3S_1 (1^3S_1 - 1^3D_1)$	J/ψ	99.93	0.07
$2^3S_1 (2^3S_1 - 2^3D_1)$	ψ'	99.90	0.10
$3^3S_1 (3^3S_1 - 3^3D_1)$	ψ'''	99.88	0.12
$1^3D_1 (1^3D_1 - 1^3S_1)$	ψ''	99.88	0.12

TABLE IV: The calculated charmonium mass spectrum as function of the number of grid points N_{P1} , N_{P1} and N_θ .

N_{p1}	N_{p2}	N_θ	η_c	J/ψ	h_c	χ_{c0}	χ_{c1}	η'_c	ψ'	ψ''	ψ'''
100	100	140	2980.602	3096.942	3526.244	3397.517	3503.552	368.3357	3760.778	3850.573	4285.415
100	100	160	2980.420	3096.951	3526.226	3397.449	3503.541	3683.153	3760.782	3850.563	4285.410
100	100	180	2980.370	3096.954	3526.221	3397.434	3503.539	3683.095	3760.784	3850.561	4285.409
100	100	200	2980.356	3096.954	3526.219	3397.431	3503.538	3683.080	3760.784	3850.560	4285.409
100	100	220	2980.353	3096.954	3526.219	3397.430	3503.538	3683.075	3760.784	3850.560	4285.409
100	100	240	2980.352	3.096954	3526.219	3397.430	3503.538	3683.074	3760.784	3850.560	4285.409
100	100	260	2980.352	3.096954	3526.219	3397.430	3503.538	3683.074	3760.784	3850.560	4285.409
100	60	200	2975.716	3097.131	3525.746	3397.449	3394.900	3503.182	3677.378	3850.232	4284.965
100	80	200	2980.218	3096.961	3526.205	3397.376	3503.530	3682.920	3760.787	3850.553	4285.405
100	100	200	2980.356	3096.954	3526.219	3397.431	3503.541	3683.538	3760.080	3850.560	4285.409
100	120	200	2980.356	3096.954	3526.219	3397.431	3503.541	3503.538	3683.080	3850.560	4285.409
50	100	200	2980.356	3096.954	3526.219	3397.429	3503.538	3683.080	3760.784	3850.558	4285.409
80	100	200	2980.356	3096.954	3526.219	3397.430	3503.538	3683.080	3760.784	3850.560	4285.409
100	100	200	2980.356	3096.954	3526.219	3397.431	3503.541	3683.080	3760.784	3850.560	4285.409

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

Appendix A: Fourier transformation of the regularized linear confining and Coulomb parts of the potential

The three-dimensional Fourier transformation of the potential $V(r)$ is defined as:

$$V(\mathbf{p}, \mathbf{p}') = \frac{1}{2\pi^2 q^2} \int_0^\infty dr r V(r) \sin qr, \quad (\text{A1})$$

where $\mathbf{q} = |\mathbf{p} - \mathbf{p}'|$. Fourier transformation of the regularized linear confining and Coulomb parts of the quark-antiquark potential is written as:

$$\begin{aligned} V(\mathbf{p}, \mathbf{p}') &= \frac{1}{2\pi^2 q^2} \left\{ \int_0^{r_c} dr r V(r) \sin qr + V(r_c) \int_{r_c}^\infty dr r \sin qr \right\} \\ &= \frac{1}{2\pi^2 q^2} \left\{ \int_0^{r_c} dr r V(r) \sin qr + V(r_c) \int_0^\infty dr r \sin qr - V(r_c) \int_0^{r_c} dr r \sin qr \right\} \\ &= \frac{1}{2\pi^2 q^2} \left\{ \int_0^{r_c} dr r V(r) \sin qr + V(r_c) \delta(\mathbf{q}) - V(r_c) \int_0^{r_c} dr r \sin qr \right\}, \end{aligned} \quad (\text{A2})$$

where potential is kept fixed at cutoff r_c . Therefore inserting the linear $V(r) = \sigma r$, and Coulomb $V(r) = f_c \alpha_s / r$, parts of quark-antiquark potential in above equation and calculation of corresponding integrals analytically, yields:

$$V(\mathbf{p}, \mathbf{p}') = \sigma \left[\delta(\mathbf{q}) r_c + \frac{1}{2\pi^2 q^4} (2 \cos(q r_c) - 2 + q r_c \sin(q r_c)) \right], \quad (\text{A3})$$

$$V(\mathbf{p}, \mathbf{p}') = f_c \alpha_s \left[\frac{\delta(\mathbf{q})}{r_c} + \frac{1}{2\pi^2 q^2} \left(1 - \frac{\sin(q r_c)}{q r_c} \right) \right]. \quad (\text{A4})$$

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